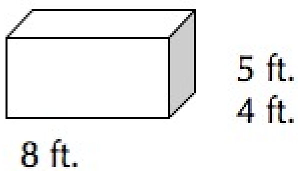
Cari's Aquarium	
Enrichment Investigation #1	
NC State Standard(s): NC.5.NBT.5 NC.5.MD.5	**Standard(s) for Mathematical Practice:** 1. Make sense of problems and persevere in solving them 2. Reason abstractly and quantifiably 3. Construct a viable argument and critique the reasoning of others. 8. Look for and express regularity in repeated reasoning
Materials Needed: Blackline master, "Cari's Aquarium" Ti-15 Calculator Graph paper/blank paper	
Instructions: 1.Distribute blackline master, "Cari's Aquarium", calculators, and graph/blank paper to students. 2. Instruct students to solve independently and then share and compare with a partner to provide feedback regarding the various possibilities. Have students check for accuracy when using the standard algorithm to multiply numbers. 3. Provide guidance with prompting and support to instruct students to think about what their tank would look like in real-life. A tank that is 40 feet long, 3 feet wide and 2 feet high would be very long, skinny and short. This tank would not be good for bigger fish, but it could be useful to watch something small that does not need a lot of space to move, such as a turtle. Monitor precision with accuracy of multiplication. 4. Allow time for students to discuss findings and ask questions with each other in a small group. Facilitate task by asking guiding questions such as "so tell me in your own words how your tank would look" or "if the ceiling is 10 feet high, how does this change how Cari might think about this last tank? 5.Present the answer tables to parts b and c to the students. Discuss the pattern observed within the possibilities as they should notice that the largest possible dimensions are used first in order to ensure no duplications. Students should also discuss the properties of multiplication. 6. If time allows, extend students' thinking to see if they can create a different shape aquarium using whole numbers for side lengths maintaining the same restrictions (240 cubic feet, not to exceed 10 feet high)	
Sources: Teacher adapted task from Illustrative Mathematics	

Cari's Aquarium

Cari is the lead architect for the city's new aquarium. All of the tanks in the aquarium will be rectangular prisms where the side lengths are whole numbers.

a. Cari's first tank is 4 feet wide, 8 feet long and 5 feet high. How many cubic feet of water can her tank hold?



b. Cari knows that a certain species of fish needs at least 240 cubic feet of water in their tank. Create 3 separate tanks that hold exactly 240 cubic feet of water.

c. In the back of the aquarium, Cari realizes that the ceiling is only 10 feet high. She needs to create a tank that can hold exactly 100 cubic feet of water. Name one way that she could build a tank that is not taller than 10 feet.

Carl's Aquarium ANSWER KEY

a. The tank can hold 160 cubic feet of water because: $\text{Length} \times \text{Width} \times \text{Height}$
$8 \text{ feet} \times 4 \text{ feet} \times 5 \text{ feet} = \text{Volume} = 160 \text{ feet cubed}$. Alternatively, the base of the tank is: $4 \text{ feet} \times 8 \text{ feet} = 32 \text{ feet}$. Therefore, $\text{Area of Base} \times \text{Height} 32 \text{ feet squared}$
$\times 5 \text{ feet} = \text{Volume} = 160 \text{ feet cubed}$.

b. There are many possible solutions to this task. The whole number factor combinations are listed below.

Length	Width	Height	Volume
240 ft.	1 ft.	1 ft.	240 ft^3
120 ft.	2 ft.	1 ft.	240 ft^3
80 ft.	3 ft.	1 ft.	240 ft^3
60 ft.	4 ft.	1 ft.	240 ft^3
60 ft.	2 ft.	2 ft.	240 ft^3
48 ft.	5 ft.	1 ft.	240 ft^3
40 ft.	6 ft.	1 ft.	240 ft^3
40 ft.	3 ft.	2 ft.	240 ft^3
30 ft.	4 ft.	1 ft.	240 ft^3
30 ft.	2 ft.	2 ft.	240 ft^3
24 ft.	10 ft.	1 ft.	240 ft^3
24 ft.	5 ft.	2 ft.	240 ft^3
20 ft.	12 ft.	1 ft.	240 ft^3
20 ft.	6 ft.	2 ft.	240 ft^3
20 ft.	4 ft.	3 ft.	240 ft^3
16 ft.	15 ft.	1 ft.	240 ft^3
16 ft.	5 ft.	3 ft.	240 ft^3

15 ft.	4 ft.	4 ft.	240 ft^3
15 ft.	8 ft.	2 ft.	240 ft^3
12 ft.	5 ft.	4 ft.	240 ft^3
12 ft.	10 ft.	2 ft.	240 ft^3
10 ft.	6 ft.	4 ft.	240 ft^3
10 ft.	8 ft.	3 ft.	240 ft^3
8 ft.	6 ft.	5 ft.	240 ft^3

c. There are many possible solutions to this task. All whole number factor combinations are listed below to ensure that students understand that they cannot simply choose any three factors that multiply to 100 $ft.^3$ without considering the restrictions on height. Cari's tank would work as long as it not higher than 10 feet tall.

Length	Width	Height	Volume
100 ft.	1 ft.	1 ft.	100 $ft.^3$
1 ft.	100 ft.	1 ft.	100 $ft.^3$
50 ft.	2 ft.	1 ft.	100 $ft.^3$
2 ft.	50 ft.	1 ft.	100 $ft.^3$
50 ft.	1 ft.	2 ft.	100 $ft.^3$
50 ft.	2 ft.	1 ft.	100 $ft.^3$
25 ft.	4 ft.	1 ft.	100 $ft.^3$
25 ft.	1 ft.	4 ft.	100 $ft.^3$
1 ft.	25 ft.	4 ft.	100 $ft.^3$
4 ft.	25 ft.	1 ft.	100 $ft.^3$
25 ft.	2 ft.	2 ft.	100 $ft.^3$
2 ft.	25 ft.	2 ft.	100 $ft.^3$

20 ft.	5 ft.	1 ft.	100 ft.^{3}
5 ft.	20 ft.	1 ft.	100 ft.^{3}
10 ft.	1 ft.	10 ft.	100 ft.^{3}
1 ft.	10 ft.	10 ft.	100 ft.^{3}
10 ft.	10 ft.	1 ft.	100 ft.^{3}
10 ft.	5 ft.	2 ft.	100 ft.^{3}
10 ft.	2 ft.	5 ft.	100 ft.^{3}
5 ft.	2 ft.	10 ft.	100 ft.^{3}
5 ft.	10 ft.	2 ft.	100 ft.^{3}
2 ft.	10 ft.	5 ft.	100 ft.^{3}
2 ft.	5 ft.	10 ft.	100 ft.^{3}
5 ft.	4 ft.	5 ft.	100 ft.^{3}
4 ft.	5 ft.	5 ft.	100 ft.^{3}
5 ft.	5 ft.	4 ft.	100 ft.^{3}

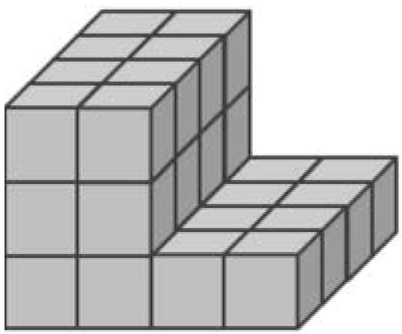
Finding Volume of Composite Figures	
Enrichment Investigation #2	
NC State Standard(s): NC.5.NBT.5 NC.5.MD.5 NC.5.OA.2	**Standard(s) for Mathematical Practice:** 1. Make sense of problems and persevere in solving them 2. Reason abstractly and quantifiably 3. Construct a viable argument and critique the reasoning of others. 6. Attend to precision. 8. Look for and express regularity in repeated reasoning
Materials Needed: Blackline master, “Decomposing Composite Figures” Crayons, markers, or colored pencils (2 different colors) Snap cubes	
Instructions: 1.Distribute blackline master, “Decomposing Composite Figures”, snap cubes, and two different colored crayons/markers/pencils to each student. This task requires students to think backwards from two different given expressions to understand how different students might decompose the same irregular solid. 2.Tell students that they will build the figure shown and explore different ways of breaking the solid into right rectangular prisms. This task relates volume to the operations of multiplication and combines operations and algebraic thinking. 3.Allow time for students to share their thinking and compare their solutions with a partner. 4.Write the following complex expressions on the board and instruct students to work in partners to explore how they can represent the same composite solid for this problem: 4 x (3 x 2 + 2) or (4 x 4 x 3) - (2 x 4 x 2). 5. Monitor student thinking as you provide feedback and pose questions. In the first example, it is very helpful to build the shape with cubes so that students can see how this solid might be broken apart. Similarly, the second example involves picturing a larger solid which might be helpful to build out with blocks. Students would then understand why a number is being subtracted out at the end.	
Sources: Teacher adapted task from Illustrative Mathematics	

Finding Volume of Composite Figures

John was finding the volume of this figure. He decided to break it apart into two separate rectangular prisms. John found the volume of the solid below using this expression:

$$(4 \times 4 \times 1) + (2 \times 4 \times 2).$$

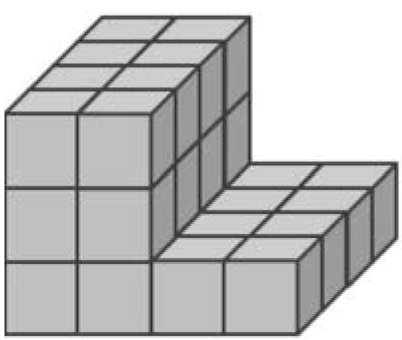
Decompose the figure into two rectangular prisms and shade them in different colors to show one way John might have thought about it.



Phillis also broke this solid into two rectangular prisms, but she did it differently than John. She found the volume of the solid below using this expression:

$$(2 \times 4 \times 3) + (2 \times 4 \times 1).$$

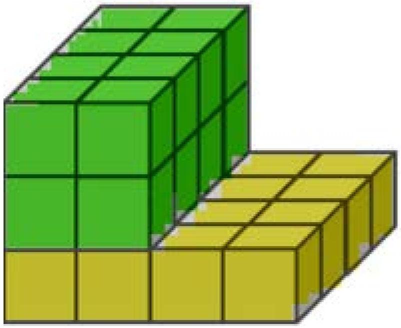
Decompose the figure into two rectangular prisms and shade them in different colors to show one way Phillis might have thought about it.



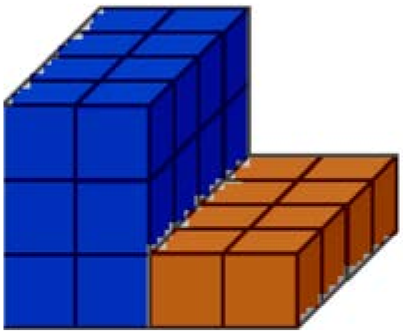
Choose one of the following expressions and explain another way to show the volume of the same shape: $4 \times (3 \times 2 + 2)$ **or** $(4 \times 4 \times 3) - (2 \times 4 \times 2)$

Finding Volume of Composite Figures ANSWER KEY

John's picture could be:



Phillis' picture could be:

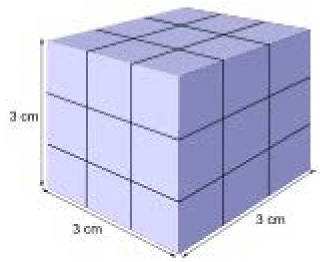


Computing Volume Complexity Problems	
Enrichment Investigation #3	
NC State Standard(s): NC.5.MD.5 NC.5.NBT.5 NC.5.NF. 4	**Standard(s) for Mathematical Practice:** 1. Make sense of problems and persevere in solving them 2. Reason abstractly and quantifiably 7. Make use of structure 8. Look for and express regularity in repeated reasoning
Materials Needed: Blackline Master, “Computing Volume Complexity Problems” Graph paper/blank paper	
Instructions: 1. Strategically group students together in partners. 2. Distribute blackline master, “Computing Volume Complexity Problems” and graph/blank paper to each student. 3. Instruct students to read problems closely and work collectively on each problem since they increase in complexity (see answer key for more information). 4. Allow time for students discuss their solutions with the group. 5. Monitor student work and probe thinking by asking questions to clarify and solidify understanding as it is connected to the standard.	
Sources: Adapted task from www.cpalms.org	

Computing Volume Complexity Problems

Problem 1

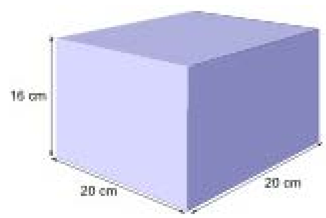
a. Amy wants to build a cube with 3 cm sides using 1 cm cubes. How many cubes does she need?



b. How many 1 cm cubes would she need to build a cube with 6 cm sides?

Problem 2

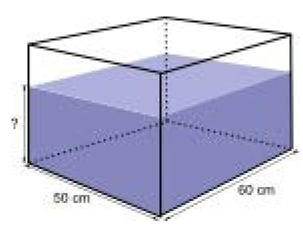
a. Amy has a fish tank shaped like a rectangular prism that is 20 cm by 20 cm by 16 cm. What is the volume of the tank?



b. If Amy only fills the tank $\frac{3}{4}$ of the way, what will be the volume of the water in the tank?

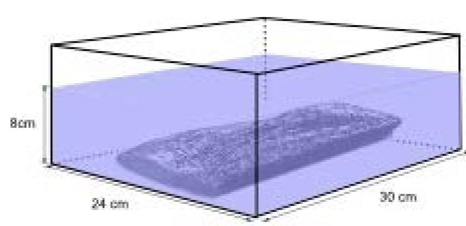
Problem 3

A rectangular tank is 50 cm wide and 60 cm long. It can hold up to $126\ \ell$ of water when full. If Amy fills $\frac{2}{3}$ of the tank as shown, find the height of the water in centimeters. (Recall that $1\ \ell = 1000\ \text{cm}^3$.)



Problem 4

A rectangular tank is 24 cm wide, and 30 cm long. It contains a stone and is filled with water to a height of 8 cm. When Amy pulls the stone out of the tank, the height of the water drops to 6 cm. Find the volume of the stone.



Computing Volume Complexity Problems ANSWER KEY

Problem 1: The purpose of this first task is to see the relationship between the side-lengths of a cube and its volume. **Solution**

a. Each single layer of cubes contains $3 \times 3 = 9$ cubes. There are 3 layers, so Amy needs $3 \times 9 = 27$ one cm cubes in all. Or, a shorter way: Amy needs $3 \times 3 \times 3 = 27$ one cm cubes.

b. Now each single layer of cubes contains $6 \times 6 = 36$ cubes. There are 6 layers, so Amy needs $6 \times 36 = 216$ one cm cubes in all. As before, we can simply compute $6 \times 6 \times 6$ to get the number of one cm cubes. Amy needs 216 one cm cubes.

Problem 2: We do away with the lines that delineate individual unit cubes (which makes it more abstract) and generalize from cubes to rectangular prisms. However, the calculations are the same as in Computing Volume Progression 1. **Solution:** Using the formula *V=lwh*

a. $V = lwh = 20 \times 20 \times 16 = 6400 \text{ cm}^3$.

b. If Amy fills the tank $\frac{3}{4}$ of the way, the height of the water in the tank will be $\frac{3}{4} \times 16 = 12$ cm, while the width and the length remain unchanged. So the volume of the water will be: $V = lwh = 20 \times 20 \times 12 = 4800 \text{ cm}^3$.

Problem 3: Here, we are given the volume and are asked to find the height. In order to do this, students must know that $1\ \ell = 1000 \text{ cm}^3$. This fact may or may not need to be included in the problem, depending on students' familiarity with the units. **Solution:** Using the formula $V = lwh$

First, find the volume of tank in cubic centimeters: $126\ell \times \frac{1000\ cm^3}{1\ell} = 126 \times 1000 \text{ cm}^3$

The height of tank is the volume divided by the length and the width:

$$\frac{126 \times 1000}{50 \times 60} = 42 \text{ cm}.$$

The height of water is $\frac{2}{3}$ the height of the tank:

$$\frac{2}{3} \times 42 = 28.$$

So the height of the water is 28 cm.

Problem 4 is based on Archimedes' Principle that the volume of an immersed object is equivalent to the volume of the displaced water. While the stone itself is an irregular solid, relating it to the displaced water in a rectangular tank means that the actual volume calculation is that of a rectangular prism. **Solution**: Using the formula *V=lwh* The change in water height is 8 cm–6 cm=2 cm. The volume of the displaced water is the product of the length, width, and change in the height of the water, and 24×30×2=1440.

The volume of the stone is the same as the volume of the displaced water, we know the stone has volume 1440 cm^3.

Pump Up the Volume (part 1)	
Enrichment Investigation #4	
NC State Standard(s): NC.5.NBT.5 NC.5.MD.5	**Standard(s) for Mathematical Practice:** 1. Makes sense and perseveres in solving problems. 2. Reasons abstractly and quantitatively. 4. Model with mathematics. 6. Attends to precision. 7. Looks for and makes use of structure.

Materials Needed:
Video
1 per group of 2 students
Product A and Product B
Centimeter measurement tools
Pencils, Paper
Index cards
Ruler
Balance or spring scale (per class)
Snap cubes (enough for each pair of students approximately 40)
Student packet including:

- Request for proposal
- Design Challenge Recording Sheet (see enrichment investigation #5 in part 2)
- Volume Data Table

Instructions:
1.Provide students with the Request Proposal from the food manufacturer, Food King, and the Design Challenge Student Planning Sheet. Instruct them to read closely. Talk about what is needed to complete the task.
2.Introduce the task and show the PBS video on the career of a packaging engineer (for background knowledge as to what a packaging engineer is). pbskids.org/designsquad/**video**/**package**-design/

3.Have students observe packaged products (that you brought in). Allow the groups to open and look at how much volume is used inside the package. Circulate and asking the following probing questions: How much space does the product take up inside the package? Is it full? Why/or why not? What 3-dimensional shapes do most packages come in?

4.While students are observing packages during last 5 minutes of engage -pass out concealed products A & B. Prompt students to identify their best design for packaging A and packaging B after they measure the volume and write dimensions on the volume/data worksheet.

5.Using their design, have students record final measurements of the dimensions of their design on their Volume Data Sheet. Remind students to round everything up and discuss why we need to do so.

6.Students will create and label final blueprint (net) of a package for their product in the blueprint sections of the Design Challenge student Recording Sheet. Students will build a model of the first layer of the volume of the base (lxw) for the package using snap cubes.

7.Once all designs have been completed, students are to independently read and complete the rationale sections of their Design Challenge Student Planning Sheet.
Sources: Teacher adapted from Engaging Mathematics

Request For Proposal

Food King

1600 Strawberry Hwy

Takis, FL 56789

Greetings Packaging Team,

Today, we are giving you a design challenge by the biggest food company, Food King, to create a package for two of their food products. Each team has been given two bags which contain Food King's new products, labeled product A and product B.

Your first task as a food package engineer is to produce packages for our new products, which are still highly top secret, therefore, you MAY NOT see the product. The new products will be delivered in a concealed, sealed container. Food King is looking for a package with a marketable design that will stand out and be recognized by our customers, along with being cost effective.

The design must contain the following:

Shape- 3-dimensional with rectangular faces

Size- cost-effective
large enough to capture consumers' attention on the shelf
give the consumer the feeling that they are getting a reasonable amount of product

Remember to include all blueprints, data sheets and a written statement as to why Food King should choose your design.

We look forward to seeing and hearing about your design!

Food King
CEO
Tim Taki

Volume Data Table

Remember to consider whether you want less material cost or more volume to attract customers. That will help you decide the size of your design.

Package for Product   A or B

Length__________

Width__________

Height__________

Base (area of largest face)___________

How many layers of cubes are needed for entire package__________

Total surface area (all the faces)__________

Volume _____________

Volume of wasted space (package's volume – product's volume)_____________

Pump Up the Product (Part 2)	
Enrichment Investigation #5	
NC State Standard(s): NC.5.MD.5 NC.5.NBT.5	**Standard(s) for Mathematical Practice:** 1. Makes sense and perseveres in solving problems. 2. Reasons abstractly and quantitatively. 4. Model with mathematics. 6. Attends to precision. 7. Looks for and makes use of structure.
Materials Needed: Paper Pencils Rulers Student packet including: • Revised request for proposal • Design challenge recording sheet (from enrichment investigation #4 part 1) • Volume data table Product A and B per group of 2 students- Centimeter measurement tools	
Instructions: 1. Introduce a new scenario and share task: Revised Request for Proposal - The Food King has decided that they would like to include product A and product B in the same package. 2.Redirect students to reread the Revised Request for Proposal from Food King. Talk about what is needed to complete the task. 3.Allow students to begin brainstorming package design options. 4.Provide students with the following: Volume Data Table Package A & B to complete. 5.Once all designs have been completed, students are to independently read and complete the rationale sections of their Design Challenge Student Planning Sheet. 6.If time permits allow whole group discussion on discoveries. Questions to ask: Discuss whose design had the least amount of area on the faces. What does this mean as to the design of the package? Whose package had the largest volume? Why did you choose to create a package with a large volume?	
Sources: Teacher adapted from Engaging Mathematics	

Food King

1600 Strawberry Hwy

Takis, FL 56789

REVISED REQUEST FOR PROPOSAL

Greetings Packaging Team,

The Food King has decided that the two new products (product A and product B) should be packaged together. Each team has been given both of Food Kings' new secret products, labeled product A and product B.

The team needs to create a blueprint of both products combined into one composite figure, along with the measurements and volume for a new package that will hold both products.

The design must address the following:

The package must have the shape of a rectangular prism.

Size-   cost-effectiveness
large enough to capture the consumer's attention
give the consumer the feeling that they are receiving a reasonable amount of product
Volume-   show the combined volume of both products (what is inside the outer package)
show the volume/measurement of the new package

Remember to include all blueprints (nets), data sheets and a written statement as to why Food King should choose your design.

We look forward to seeing and hearing about your design!

Food King
CEO
Tim Taki

Volume Data Table Product A & B

Product A

Length___________

Width___________

Height___________

Volume____________

Product B

Length_________

Width__________

Height__________

Volume ____________

New Package (volume composite package A and B)

Length________ Width_________ Height ________ Volume ___________

Volume of wasted space (package's volume – product's volume)______________

Name: ___________________________ Date:_________________

Design Challenge Student Recording Sheet

Problem Write a summary of the Request for Proposal in your own words.

____

____

____

____

Brainstorm Design two different packages for each product. Make sure to label your designs.

A1

A2

B1

B2

How are you determining the dimensions of your package?

____

____

What does volume measure?

____

____

Blueprint (net)

Draw a final blueprint for each package. Remember to include dimensions and labels for each.

A

B

Rationale Why should Food King choose your packages for their food products? Explain.

Blueprint (net)

Draw a final blueprint for each package. Remember to include dimensions and labels for each.

A & B inside

New design

Rationale

Why should Food King choose your packages for their food products? Explain.

____

____

____

____

____